## Maximizing Diversity

## I'm off to find multiple optima

## Diversity Measure



## Diversity Measure

Takes into account $f(x)$ to measure how far two points are from each other.

If two points are at the peak of adjacent optima, the distance will be maximal (maybe).

If they are at the same side of the same optimum, the distance will be very small.

## Maximizing Diversity

Maximize the sum of pairwise distances from a population.

It aims at finding solutions that are located at different peaks around a given region.

## But...



## Maximizing Diversity

Maximize the sum of distances from each solution of the population to its nearby (Euclidean) solution.

It aims at spreading the solutions among the optima of a given region.

## Maximizing Diversity



## Simple Test

Starting at the center of the bounds:

- sample a random direction
- maximize diversity following this direction and the opposite


## Simple Test



The middle points are the maxima and minima...

## Simple Test


...or the middle points are the minima and the extreme are the maxima.

## Simple Test

After this procedure, repeat:

- sample one local optima
- calculate its radius of attraction (distance between maxima and minima)
- sample new solutions centered by this optima and separated by multiples of this radius
- apply previous maximization to these points


## Simple Test

Rationale: if the optima are spread uniformly on the search space, this sampling will have a high success on hitting other optima.

If not, the diversity maximization will lead them to the closest local optima.

## Results so far...

| Function | \# found |  | \# of optima | Diversity |
| ---: | ---: | ---: | ---: | ---: |
| NEA2 |  |  |  |  |
| 1 | 2 | 2 | 1 | 1 |
| 2 | 5 | 5 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 4 | 4 | 1 | 1 |
| 5 | 2 | 2 | 1 | 1 |
| 6 | 18 | 18 | $\mathbf{N}$ | 0.48 |
| 7 | 15 | 36 | $\mathbf{0 . 4 2}$ | 0.16 |
| 8 | 0 | 81 | 0 | 0 |
| 9 | 38 | 216 | $\mathbf{0 . 1 8}$ | 0 |
| 10 | 12 | 12 | 1 | 1 |

## Conclusion

There's still a long way to go...

When the function has few optima with the same characteristics, it does a pretty good job.

If the function has many optima, it will try to find them all, may take a long time.

## Future Directions

This is still an early work, I still should try:

- introduce this maximization into well-known meta-heurístics (but it is not as simple as a objective function change)
- verify some other hypothesis


## Images

## created using coLaboratory:

https://drive.google.com/file/d/OB2w0UKm5DHxiX21YUVFLeVE0QzA/edit?usp =sharing

## Results so far...

| Function | \# found |  | \# of optima | Skipy |
| ---: | ---: | ---: | ---: | ---: |
|  | N |  | NEA2 |  |
| 1 | 2 | 2 | 1 | 1 |
| 2 | 5 | 5 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 |
| 4 | 4 | 4 | 1 | 1 |
| 5 | 2 | 2 | 1 | 1 |
| 6 | 17 | 18 | 1 | 0.48 |
| 7 | 35 | 36 | $\mathbf{1}$ | 0.16 |
| 8 | 1 | 81 | 0 | 0 |
| 9 | 82 | 216 | $\mathbf{0 . 0 2}$ | 0 |
| 10 | 4 | 12 | 0.58 | 1 |

## Results so far...

| Function | \# found |  | \# of optima | Diversity |
| ---: | ---: | ---: | ---: | ---: |
| NEA2 |  |  |  |  |
| 11 | 1 | 6 | 0.17 | 0 |
| 12 | 0 | 8 | 0 | 0 |
| 13 | 1 | 6 | 0.17 | 0 |
| 14 | 1 | 6 | 0.17 | 0 |
| 15 | 0 | 8 | 0 | 0 |
| 16 | 0 | 6 | 0 | 0 |
| 17 | 0 | 8 | 0 | 0 |
| 18 | 0 | 6 | 0 | 0 |
| 19 | 0 | 6 | 0 | 0 |
| 20 | 0 | 8 | 0 | 0 |

